

Dynamics of an exploited prey-predator fish species with Beddington-DeAngelis functional response

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Abstract: This paper deals with the combined harvesting of a prey-predator fishery with the Beddington-DeAngelis functional response [1,2]. The Beddington-DeAngelis functional response is similar to the Holling type II functional response but contains one extra term describing mutual interference by the predators. Here the prey species obeys the logistic law of growth [3]. The existence of the steady states of the dynamical system is discussed. Both local and global stability of all possible steady states are studied. It is also examined whether the system possesses any limit cycle. Existence of bionomic equilibrium is also examined. All the results are illustrated with the help of a numerical example.

Keywords: prey-predator fishery, Beddington-DeAngelis functional response, biotechnical productivity, local stability, global stability, limit cycle, bionomic equilibrium.

1. INTRODUCTION:

In nature there are many interactions between different species. One of the common interactions is that of prey and predator. A predator species kills the prey species and the prey species is killed and eaten by the predator. In the fish species, prey-predator interaction is a very common problem for extinction of the prey fish population. Moreover, common harvesting of prey-predator species is a global problem for extinction of both the species. For existence of the biological equilibrium of the fish populations, the Government or any private agencies should regulate the harvesting effort in order to control overexploitation of the fish species.

Many researchers developed different types of exploited prey-predator models and discussed the biological equilibrium of the populations and their dynamical behaviour. Saha Ray and Chaudhuri [4] discussed a Lotka-Volterra prey-predator model with harvesting and environmental perturbations. Bionomic exploitation of a Lotka-Volterra prey-predator system was studied by Chaudhuri and Saha Ray [5]. The optimal policy for combined harvesting of a prey-predator community was discussed by Mesterton-Giobbons [6]. Chaudhuri and Pradhan [7] developed a model of combined harvesting of a prey-predator fishery with low predator density and the prey species obeying the logistic law of growth. Kar [8] developed a prey-predator model with stage-structure for predator and selective harvesting of prey species.

Holling type I ($f(x) = ax$), II ($f(x) = \frac{ax}{x+\beta}$) and III ($f(x) = \frac{ax^2}{x^2+\beta}$) [9] functional responses are the most popularly used functional responses for the pre-predator models. But all

these functional responses are prey dependent and so in some situation there may be an unrealistic population dynamics of the prey or predator. This needs a prey-predator dependent functional responses $f(x, y)$. The Beddington-DeAngelis type functional response [1,2], $f(x, y) = \frac{ax}{b+cx+y}$ ($b > 0, c > 0, a > 0$) is one of the more realistic functional responses. It is similar to the Holling type II functional response but has an extra term y in the denominator of $f(x, y)$ which models mutual interference between the predators.

In this paper the dynamics of an exploited prey-predator fishery with Beddington-DeAngelis type functional response is studied. The dynamical behaviour of the system is first studied. It is shown that the boundary and the interior equilibrium points exist under certain conditions which are amenable to interpretations relevant to the model. Both local and global stability of the steady states are then discussed. Existence of limit cycles is also examined. The Bionomic equilibrium of the system is discussed next. A numerical example is taken to illustrate the results.

2. THE MATHEMATICAL MODEL:

A general prey-predator model can be described as

$$\left. \begin{aligned} \frac{dx(t)}{dt} &= i(x) - pf(x, y)y \\ \frac{dy(t)}{dt} &= qf(x, y)y - m(y) \end{aligned} \right\} x(0) > 0, y(0) > 0 \quad (1)$$

where $x(t)$ and $y(t)$ be the population density of the prey and the predator species respectively at any time t , $i(x)$ is the intrinsic growth rate of the prey, $f(x, y)$ is the functional response which represents per capita predator feeding rate and $m(y)$ is the mortality rate of the predator.

Here we consider a prey-predator model of fish species where the intrinsic growth rate of the prey obeys the logistic growth function [3] i.e., $i(x) = rx\left(1 - \frac{x}{k}\right)$ and the mortality rate of the predator is dy . The Beddington-DeAngelis [1,2] type functional response

$f(x, y) = \frac{\alpha x}{b+cx+y}$ ($b > 0, c > 0, \alpha > 0$) is considered here which is more realistic than the functional responses Holling type I,II,III [9] as described by D.T. Dimitrov and H.V. Kojouharov [10].

Now, $f(x, y) > 0, \frac{\partial f}{\partial x} = \frac{(b+y)\alpha}{(b+cx+y)^2} > 0,$

$$\frac{\partial f}{\partial y} = -\frac{\alpha x}{(b+cx+y)^2} < 0 \quad \forall x, y \in R_2^+ = \{(x, y), x > 0, y > 0\}$$

imply that in the positive quadrant of R_2 , per capita predator feeding rate is always positive and is an increasing function of x and decreasing function of y . Biological implication of these results is, in the increase of the prey species the predator feeding rate also increases and in the increase of the predator species the feeding rate decreases due to interspecies competitions.

After the above consideration, the system (1) becomes

$$\left. \begin{aligned} \frac{dx(t)}{dt} &= rx\left(1 - \frac{x}{k}\right) - \frac{\alpha xy}{b+cx+y} \\ \frac{dy(t)}{dt} &= \frac{exy}{b+cx+y} - dy \end{aligned} \right\} x(0) > 0, y(0) > 0 \quad (2)$$

where r, k, a, b, c, d, e are all positive constants.

- Here r = intrinsic growth rate of the prey species,
- k = environmental carrying capacity of the prey population,
- a = capturing rate of the prey by the predator,
- e = maximal predator growth rate,
- d = per capita death rate of the predator.

We now assume that both the species are subjected to a combined harvesting effort E .

Therefore, the system of equations (2) becomes

$$\left. \begin{aligned} \frac{dx(t)}{dt} &= rx\left(1 - \frac{x}{k}\right) - \frac{\alpha xy}{b+cx+y} - q_1Ex \\ \frac{dy(t)}{dt} &= \frac{exy}{b+cx+y} - dy - q_2Ey \end{aligned} \right\} x(0) > 0, y(0) > 0 \quad (3)$$

where q_1 and q_2 are catchability coefficients of the prey and predator species respectively. Now we intend to study the characteristics of the exploited prey-predator fish species described by the dynamical system (3) and also some economic implications of the harvesting policy.

3. DYNAMICAL BEHAVIOUR:

A) Steady states

$P_0(0,0)$ is the trivial steady state of the system of equations (3). If $x = 0$, then $\frac{dy}{dt} = -(d + q_2)y$. This implies $y(t) = C_0 e^{-(d+q_2)t}$ i.e. $y(t) \rightarrow 0$ as $t \rightarrow \infty$. This implies that in absence of prey, the predator species exponentially extinct. Biologically it is true that a predator species cannot live without prey species.

Again $P_1(\bar{x}, 0)$ is the boundary steady state of the system (3) if $E < \frac{r}{q_1}$ where $\bar{x} = k\left(1 - \frac{q_1 E}{r}\right)$. (4)

In fishery literature the ratio of the biotic potential r to the catchability coefficient q_1 is called the BTP (biotechnical productivity) [11]. If $E = \frac{r}{q_1}$, then $\bar{x} = 0$ i.e. in absence of predator, if the harvesting effort is equal to the prey BTP, the prey species extinct and P_1 coincides with the trivial steady state P_0 . Therefore, in order to exist the boundary steady state $P_1(\bar{x}, 0)$, the effort level should be less than the prey BTP.

$P_2(x^*, y^*)$ is the interior equilibrium point of the dynamical system (3) where x^* and y^* are given by

$$x^* = \frac{kB + \sqrt{k^2 B^2 + 4kC}}{2} > 0 \quad \forall B, C \quad (5)$$

$$y^* = \left(\frac{e}{d+q_2 E} - c\right) x^* - b \quad (6)$$

$$\text{where } B = \left(1 - \frac{a}{r} - \frac{q_1 E}{r}\right) + \frac{ac(d+q_2 E)}{re} \quad (7)$$

$$\text{and } C = \frac{ab(d+q_2 E)}{re} > 0 \quad (8)$$

The necessary condition for $y^* > 0$ is

$$e > c(d + q_2 E) = L(> 0) \text{ (say)} \quad (9)$$

So for the existence of the interior equilibrium point of the system (3) one of the necessary conditions is that the maximal growth rate of the predator is greater than L . Condition (9) is equivalent to $E < \frac{e-cd}{cq_2} = M(\text{say}) > 0$ by (8). (10)

Therefore, M is the maximum effort level for existence of the interior equilibrium point of the system (3). But the above two conditions (9) and (10) are not sufficient for existence of the interior equilibrium point of the dynamical system (3). For $y^* > 0$, it is also necessary that

$$x^* > \frac{b(d+q_2 E)}{e-c(d+q_2 E)} = m \text{ (say)}. \quad (11)$$

Therefore, m is the greatest lower bound (infimum) of the prey population for existence of the non-trivial steady state of the predator species. If prey species is below the level m ,

then $\frac{dy}{dt} < 0$ and the predator species exponentially dies out. Thus we have the following theorem:

Theorem 1: The necessary conditions for existence of the non-trivial equilibrium point $P_2(x^*, y^*)$ of the dynamical system (3) are

$$(i) e > c(d + q_2E) = L \quad (ii) E_{max} = \frac{e-cd}{cq_2} = M \quad (iii) \inf\{x(t)\} = \frac{b(d+q_2E)}{e-c(d+q_2E)} = m.$$

B) Local stability

The variational matrix of the dynamical system (3) is

$$V(x, y) = \begin{pmatrix} J_{11} & J_{12} \\ J_{21} & J_{22} \end{pmatrix} \text{ where}$$

$$J_{11} = \frac{\partial}{\partial x} \left(\frac{dx}{dt} \right) = r - \frac{2rx}{k} - \frac{ay}{b+cx+y} + \frac{acxy}{(b+cx+y)^2} - q_1E,$$

$$J_{12} = \frac{\partial}{\partial y} \left(\frac{dx}{dt} \right) = -\frac{(b+cx)ax}{(b+cx+y)^2},$$

$$J_{21} = \frac{\partial}{\partial x} \left(\frac{dy}{dt} \right) = \frac{ey(b+y)}{(b+cx+y)^2},$$

$$J_{22} = \frac{\partial}{\partial y} \left(\frac{dy}{dt} \right) = \frac{ex}{b+cx+y} - \frac{exy}{(b+cx+y)^2} - d - q_2E.$$

$$\text{At } P_0(0,0), V(0,0) = \begin{pmatrix} r - q_1E & 0 \\ 0 & -(d + q_2E) \end{pmatrix}.$$

Therefore, the eigen values of $V(0,0)$ are $r - q_1E$ and $-(d + q_2E)$.

If the prey BTP $\frac{r}{q_1} > E$, then the trivial steady state $P_0(0,0)$ is a saddle point. On the other hand if the prey BTP $\frac{r}{q_1} < E$ then the trivial steady state $P_0(0,0)$ is a stable node. But in this case non-trivial steady state $P_1(\bar{x}, 0)$ does not exist. Therefore, for existence of the prey species in absence of predator, the effort level for harvesting of the prey species must be less than the prey BTP ($\frac{r}{q_1}$) and so the trivial steady state $P_0(0,0)$ of the system (3) is always a saddle point.

$$\text{At } P_1(\bar{x}, 0), V(\bar{x}, 0) = \begin{pmatrix} r - \frac{2r\bar{x}}{k} - q_1E & -\frac{a\bar{x}}{b+c\bar{x}} \\ 0 & \frac{e\bar{x}}{b+c\bar{x}} - d - q_2E \end{pmatrix}.$$

Therefore, the eigen values of $V(\bar{x}, 0)$ are $-(r - q_1E) < 0$ since $E < \frac{r}{q_1}$ and $\frac{ek(r-q_1E)}{br+ck(r-q_1E)} - d - q_2E$.

So $P_1(\bar{x}, 0)$ is a saddle point or a stable node according as $\frac{ek(r-q_1E)}{br+ck(r-q_1E)} - d - q_2E > 0$ or < 0 . (12)

When $e < c(d + q_2E)$, then $\frac{ek(r-q_1E)}{br+ck(r-q_1E)} - d - q_2E < 0$ and then $P_1(\bar{x}, 0)$ is a stable node. But in that case the interior equilibrium point $P_2(x^*, y^*)$ does not exist by theorem 1.

$$\text{At } P_2(x^*, y^*), V(x^*, y^*) = \begin{pmatrix} \frac{acx^*y^*}{A} - \frac{rx^*}{k} - \frac{ax^*(b+cx^*)}{A} \\ \frac{ey^*(b+y^*)}{A} - \frac{ex^*y^*}{A} \end{pmatrix}$$

where $A = (b + cx^* + y^*)^2$.

$$TrV(x^*, y^*) = \frac{acx^*y^*}{A} - \frac{rx^*}{k} - \frac{ex^*y^*}{A} \tag{13}$$

$$detV(x^*, y^*) = \frac{erx^{*2}y^*}{kA} + \frac{eabx^*y^*}{A^2} (b + y^* + cx^*) > 0 \tag{14}$$

If $TrV(x^*, y^*) < 0$, then the non-zero equilibrium point $P_2(x^*, y^*)$ is stable. Otherwise it is unstable. From (13) it is clear that $e > ac$ is the sufficient condition for the steady state $P_2(x^*, y^*)$ is locally stable.

If $d > a$ i.e. mortality rate of predator is greater than the prey capturing rate by the predator, then by of theorem-1 (i), $P_2(x^*, y^*)$ is always stable. But if $d < a$, then $E > \frac{a-d}{q}$ is the sufficient condition for local stability of the point $P_2(x^*, y^*)$. $P_2(x^*, y^*)$ will be stable node or stable focus according as the characteristic roots of the variational matrix $V(x^*, y^*)$ are real negative or complex conjugate with negative real parts. Thus we have the following theorem:

Theorem 2: Whenever the non-trivial equilibrium point $P_2(x^*, y^*)$ of the system of equations (3) exists then (i) if the mortality rate of predator is greater than the prey capturing rate ($d > a$), then $P_2(x^*, y^*)$ is always locally stable and (ii) if $d < a$, then $P_2(x^*, y^*)$ is locally stable if $E > \frac{a-d}{q}$.

Remark: Condition (ii) of theorem-2 is the sufficient condition not necessary.

C) Limit cycles

Theorem 3: If the harvesting effort is less than or equal to the prey BTP ($E \leq \frac{r}{q_1}$), then the system of equations (3) does not possess limit cycles in the region $R_2^+ = \{(x, y); x > 0, y > 0\}$.

Proof: We now examine the possibility of existence of a limit cycle of the non-linear system (3) by using Bendixon-Dulac test.

Let us consider a transformation $dt = (b + cx + y)d\theta$. Denoting new argument θ with t again, the system (3) can be written as

$$\frac{dx}{dt} = rx \left(1 - \frac{x}{k} \right) (b + cx + y) - axy - q_1Ex(b + cx + y) = f(x, y)$$

$$\frac{dy}{dt} = exy - dy(b + cx + y) - q_2Ey(b + cx + y) = g(x, y).$$

Define a Dulac function $B(x, y) = x^{-1}y^{-1}$.

Now, $\frac{\partial}{\partial x}(Bf) + \frac{\partial}{\partial y}(Bg) = -\frac{1}{k}ry^{-1}(b + cx + y) +$
 $cray^{-1}\left(1 - \frac{x}{k}\right) - q_1cEy^{-1} - dx^{-1} - q_2Ex^{-1}.$
 $= -\frac{br}{ky} - \frac{2crx}{ky} - \frac{r}{k} - \frac{d}{x} - \frac{q_1E}{x} - \frac{c}{y}(q_1E - r) < 0$ if
 $E \leq \frac{r}{q_1} \forall x, y \in R_2^+$.

Hence the expression $\frac{\partial}{\partial x}(Bf) + \frac{\partial}{\partial y}(Bg)$ does not change the sign in R_2^+ and so the system (3) does not possess the limit cycle in R_2^+ .

Remark: $E \leq \frac{r}{q_1}$ is the sufficient condition for non existence of the limit cycle in R_2^+ but not necessary.

D) Global stability

We have already noted that the dynamical system (3) has a unique positive non-trivial equilibrium point at $P_2(x^*, y^*)$ provided the conditions of theorem-1 hold. We now examine the global stability of $P_2(x^*, y^*)$ of the system (3). For the fixed environmental carrying capacity k for the prey species and for existence of the steady state $P_2(x^*, y^*)$, the prey species must be bounded in the range $m < x < k$. Also for the natural mortality, effects of harvesting and crowding effects the predator species must has an upper bound. So the solutions of the dynamical (3) are uniformly bounded in a finite region Ω_1 in the positive quadrant of x - y plane.

We define a Lyapunov function

$L(x, y) = x - x^* - x^* \ln\left(\frac{x}{x^*}\right) + \mu \left\{ y - y^* - y^* \ln\left(\frac{y}{y^*}\right) \right\}$ where μ is a suitable positive constant to be determined in the subsequence steps. $L(x, y)$ is a positive definite function in the region Ω_1 except at $P_2(x^*, y^*)$ where it vanishes.

Here $L(x^*, y^*) = 0$

and $\lim_{(x,y) \rightarrow (0,0)} L(x, y) = \lim_{(x,y) \rightarrow (\infty, \infty)} L(x, y) = \infty$. The time derivative of $L(x, y)$ along the solution of (3) is

$$\begin{aligned} \frac{dL}{dt} &= \left(\frac{x-x^*}{x}\right) \frac{dx}{dt} + \mu \left(\frac{y-y^*}{y}\right) \frac{dy}{dt} \\ &= (x - x^*) \left\{ r \left(1 - \frac{x}{k}\right) - \frac{ay}{b + cx + y} - q_1E \right\} \\ &\quad + \mu(y - y^*) \left\{ \frac{ex}{b + cx + y} - d - q_2E \right\} \\ &= (x - x^*) \left\{ r \left(1 - \frac{x}{k}\right) - \frac{ay}{b + cx + y} - r \left(1 - \frac{x^*}{k}\right) \right. \\ &\quad \left. + \frac{ay^*}{b + cx^* + y^*} \right\} \\ &\quad + \mu(y - y^*) \left\{ \frac{ex}{b + cx + y} - \frac{ex^*}{b + cx^* + y^*} \right\} \end{aligned}$$

$$\begin{aligned} &= (x - x^*) \left\{ -\frac{r}{k}(x - x^*) - \frac{ab(y - y^*) + ac(x^*y - y^*x)}{(b + cx + y)(b + cx^* + y^*)} \right\} \\ &\quad + \mu(y - y^*) \left\{ \frac{eb(x - x^*) - e(x^*y - y^*x)}{(b + cx + y)(b + cx^* + y^*)} \right\} \\ &= -\frac{r}{k}(x - x^*)^2 - \frac{ab(x - x^*)(y - y^*)}{(b + cx + y)(b + cx^* + y^*)} - \frac{ac(x - x^*)(x^*y - y^*x)}{(b + cx + y)(b + cx^* + y^*)} \\ &\quad + \frac{\mu eb(x - x^*)(y - y^*)}{(b + cx + y)(b + cx^* + y^*)} - \frac{\mu e(y - y^*)(x^*y - y^*x)}{(b + cx + y)(b + cx^* + y^*)} \\ &= -\frac{r}{k}(x - x^*)^2 - \frac{ac(x - x^*)(x^*y - y^*x)}{(b + cx + y)(b + cx^* + y^*)} - \frac{a(y - y^*)(x^*y - y^*x)}{(b + cx + y)(b + cx^* + y^*)} \end{aligned}$$

for $\mu = \frac{a}{e} > 0$.

$$= -\frac{r}{k}(x - x^*)^2 - \frac{a(x^*y - y^*x)(cx + y - cx^* - y^*)}{(b + cx + y)(b + cx^* + y^*)} < 0$$

$\forall (x, y) \in \Omega_2 \cap \Omega_3$ where

$\Omega_2 = \{(x, y) : x^*y - y^*x \geq 0, cx + y - cx^* - y^* \geq 0\}$ and

$\Omega_3 = \{(x, y) : x^*y - y^*x \leq 0, cx + y - cx^* - y^* \leq 0\}$.

Also $\frac{dL}{dt} = 0$ at $P_2(x^*, y^*)$.

Hence by Lassalle's invariance principle [12], $P_2(x^*, y^*)$ is globally asymptotically stable for all $(x, y) \in \Omega$ where $\Omega = \Omega_1 \cap \Omega_2 \cap \Omega_3$.

4. BIONOMIC EQUILIBRIUM:

The term *bionomic equilibrium* is an amalgamation of the concepts of *biological equilibrium* as well as *economic equilibrium*. We have already find the *biological equilibrium* which are given by $\dot{x} = \dot{y} = 0$. The *economic equilibrium* is said to be achieved when the total revenue obtained by selling the harvested biomass (TR) equals the total cost for the effort devoted to harvesting (TC).

Let $c_p =$ fishing cost per unit effort,

$p_1 =$ price per unit bio-mass of the prey species and

$p_2 =$ price per unit bio-mass of the predator species.

The *economic net revenue* at any time is given by

$\pi(x, y, E) = TR - TC = p_1q_1Ex + p_2q_2Ey - c_pE$ (15)

The bionomic equilibrium $(x_\infty, y_\infty, E_\infty)$ is given as a solution of the $\dot{x} = \dot{y} = \pi = 0$. i.e.

$r \left(1 - \frac{x}{k}\right) - \frac{ay}{b + cx + y} - q_1E = 0$ (16)

$\frac{ex}{b + cx + y} - d - q_2E = 0$ (17)

$p_1q_1x + p_2q_2y - c_p = 0$ (18)

Eliminating E from (16) and (17), we have

$q_2r \left(1 - \frac{x}{k}\right) - \frac{aq_2y}{b + cx + y} - \frac{eq_1x}{b + cx + y} + q_1d = 0$ (19)

From (18), we have $y = \frac{c_p - p_1q_1x}{p_2q_2}$ (20)

Eliminating y from (19) and (20), we have

$A_0x^2 - A_1x + A_2 = 0$ (21)

where $A_0 = \frac{p_1 q_1^2 r}{k p_2 q_2} - \frac{q_1 r c}{k}$,

$$A_1 = c q_1 r - \frac{p_1 q_1^2}{p_2 q_2} - \frac{b q_1 r}{k} - \frac{q_1 r c_p}{k p_2 q_2} + \frac{a p_1 q_2}{p_2} - e q_1 + q_1 d c - \frac{p_1 q_1^2 d}{p_2 q_2},$$

$$A_2 = q_1 b r + \frac{q_1 c_p}{p_2 q_2} - \frac{a c_p}{p_2} + q_1 b d + \frac{q_1 c_p d}{p_2 q_2}.$$

If x_∞ be the positive root of the equation (21), then from (20) we have $y_\infty = \frac{c_p - p_1 q_1 x_\infty}{p_2 q_2}$ and from (17) we have

$$E_\infty = \frac{1}{q_2} \left(\frac{e x_\infty}{b + c x_\infty + y_\infty} - d \right). \text{ Therefore, } (x_\infty, y_\infty, E_\infty) \text{ be the bionomic equilibrium provided } x_\infty < \frac{c_p}{p_1 q_1} \text{ and } d < \frac{e x_\infty}{b + c x_\infty + y_\infty}.$$

5. NUMERICAL EXAMPLE:

Let $r = 5, k = 100, a = 0.3, b = 1, c = 2, d = 0.25, e = 4, q_1 = 0.05, q_2 = 0.07, E = 20$ in appropriate units.

Here the harvesting effort $E (= 20)$ is less than the prey BTP ($\frac{r}{q_1} = 100$), so the boundary equilibrium exists and $P_1(80, 0)$ is the boundary equilibrium of the system (3).

Using the above parameter values we have, $L = c(d + q_2 E) = 3.3, e = 4$, and $E_{max} = 25$. So all the necessary conditions for existence of the non-trivial equilibrium point for the prey species are satisfied. From (7) and (8) we have $B = 0.7895$ and $C = 0.02475$ respectively. Using these values of B and C , from (5) we have $x^* = 78.98$. From (10), $m = \inf x(t) = \frac{b(d + q_2 E)}{e - c(d + q_2 E)} = 2.358 < x^*$, so, y^* exists and from (6), we have $y^* = 32.51$. Therefore, $P_2(78.98, 32.51)$ is the non-trivial equilibrium point of the dynamical system (3).

Since the effort is less than the prey BTP, so the trivial steady state $P_0(0, 0)$ is a saddle point of the system (3). From (11) we have $\frac{ek(r - q_1 E)}{br + ck(r - q_1 E)} - d - q_2 E = 0.338 > 0$. So the boundary equilibrium $P_1(80, 0)$ of the system (3) is a saddle point.

From (12) and (13) we have, $TrV(78.98, 32.51) = -4.187128$ and $detV(78.98, 32.51) = 1.106756$. Therefore, the characteristic roots of the variational matrix $V(78.98, 32.51)$ are the roots of the equation $\lambda^2 + 4.187128\lambda + 1.106756 = 0$. Sum of the roots of this equation is negative and the product of the roots is positive and the discriminant of this equation is $13.431 > 0$. So all the characteristic roots of the variational matrix $V(78.98, 32.51)$ are real and negative. Therefore the non-trivial equilibrium point $P_2(78.98, 32.51)$ of the dynamical system (3) is a stable node. Since the effort is less than the prey BTP, so by theorem 2, the dynamical system (3) does not possess any limit cycle in the region R_2^+ and so $P_2(78.98, 32.51)$ is globally asymptotically stable node in R_2^+ .

Let the fishing cost per unit effort is $c_p = 30$, the price per unit biomass of the prey and predator species are $p_1 = 3$ and $p_2 = 6$ respectively in appropriate units. Therefore, the equation (18) becomes $x^2 - 32.167x - 785.68 = 0$. The only positive root of this equation is $x_\infty = 48.40$. For this

value of x_∞ , we have from (19), $y_\infty = 54.14$ and $E_\infty = 14.63$, by (20). Therefore, $(48.40, 54.14, 14.63)$ be the bionomic equilibrium of the system (3).

6. CONCLUSION:

In this paper, the Beddington-DeAngelis functional response is considered instead of frequently used Holling types I, II, III functional responses. This functional response is more realistic due to presence of one extra term y in the denominator of the functional response which describes mutual interference by the predators. Biological interpretation of this functional response is, in the increase of the prey species the predator feeding rate increases and in the increase of the predator species the feeding rate decreases due to interspecies competitions. It has been shown that the boundary equilibrium of the dynamical system exists when the effort less than the prey BTP and the interior equilibrium point of the system exists when the harvesting effort less than a certain level. So considering the other parameter values the regulatory agencies determine the maximum effort level for existence of the non-trivial equilibrium point. It is also proved that when the prey population is below a certain level then the predator population exponentially dies out.

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